Imperfections in the sideband responsive behaviour that corrupt the detection

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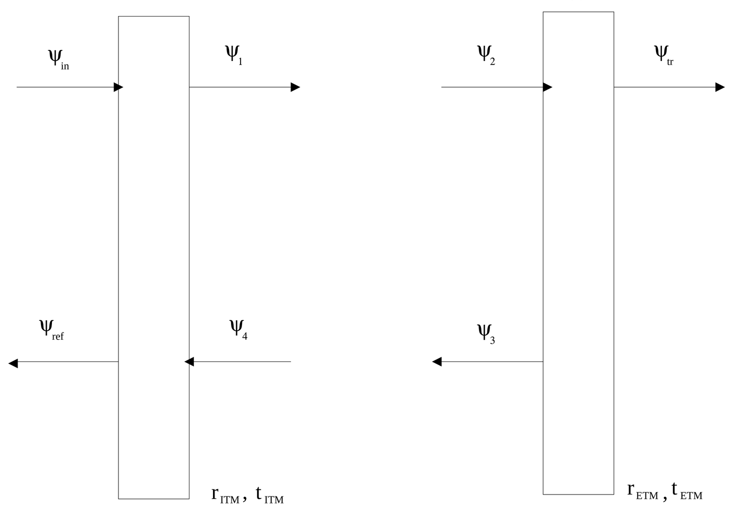
A gravitational wave is measurable as a phase change in the electromagnetic field that is reflected out of an **optical resonator driven by monochromatic light** (carrier field)

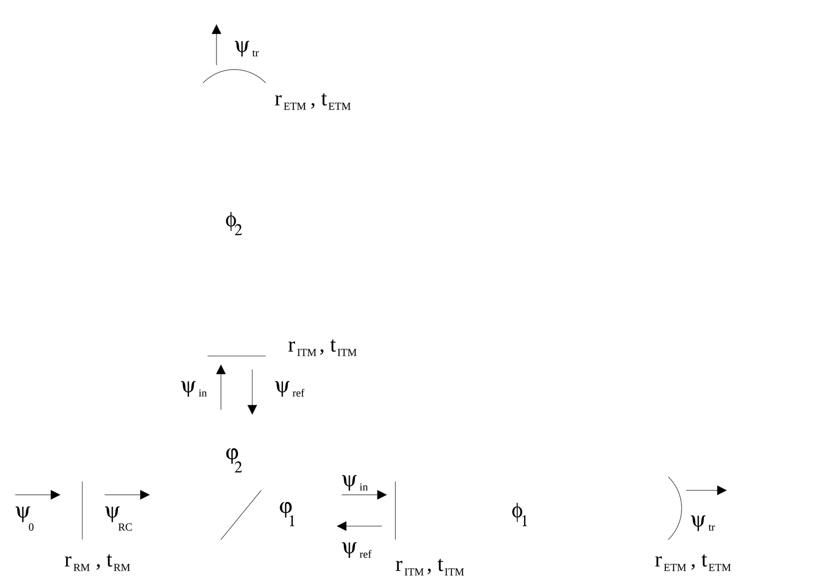
$$r_{eff}(\phi) = \frac{r_{ITM} - r_{ETM}e^{i\phi}}{1 - r_{ITM}r_{ETM}e^{i\phi}}$$

$$r_{eff}(\phi_{gw}) \overset{r_{ETM}\to 1}{\longrightarrow} \exp\frac{2i\mathcal{F}\phi_{gw}}{\pi}$$



$$\mathcal{F} \equiv \frac{\pi\sqrt{r_{ITM}r_{ETM}}}{1 - r_{ITM}r_{ETM}}$$

The detector can be designed in order to make the application of the basic idea work easier and a more complex scheme is used for this purpose



$$t_1 = \frac{1}{\sqrt{2}}\frac{t_{RM}e^{i\varphi_1/2}}{[1 - r_{RM}(r_{eff}(\phi_1)e^{i\varphi_1} + r_{eff}(\phi_2)e^{i\varphi_2})/2]}$$

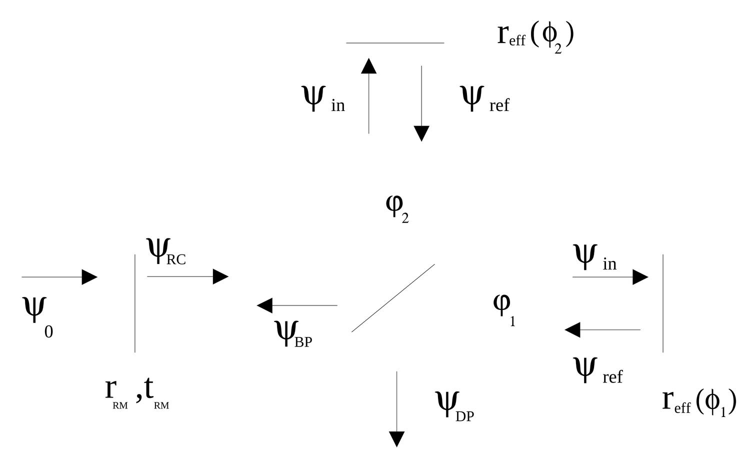
$$t_2 = \frac{1}{\sqrt{2}}\frac{t_{RM}e^{i\varphi_2/2}}{[1 - r_{RM}(r_{eff}(\phi_1)e^{i\varphi_1} + r_{eff}(\phi_2)e^{i\varphi_2})/2]}$$

In an ideal interferometer the amplitude of the field at the dark port is proportional to the phase variation induced by the gravitational wave

$$\Psi_{DP} = \frac{1}{2}[r_{eff}(\phi_1)e^{i\varphi_1} - r_{eff}(\phi_2)e^{i\varphi_2}]\Psi_{RC}$$

at the working point

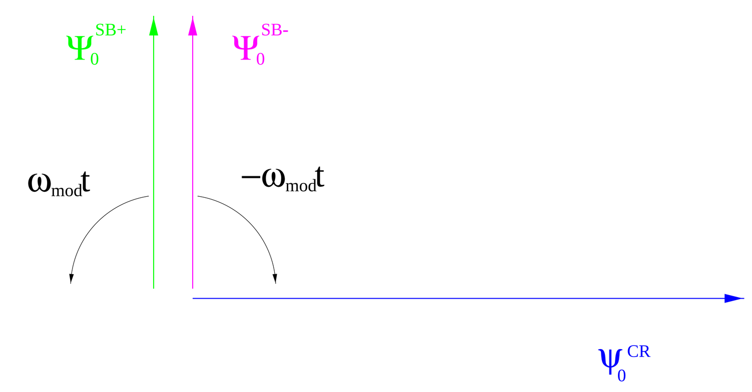
$$\phi_1 = \phi_2 = 0 \qquad \varphi_1 = \varphi_2 = 0$$



$$\phi_1 = -\phi_2 = \phi_{gw}$$

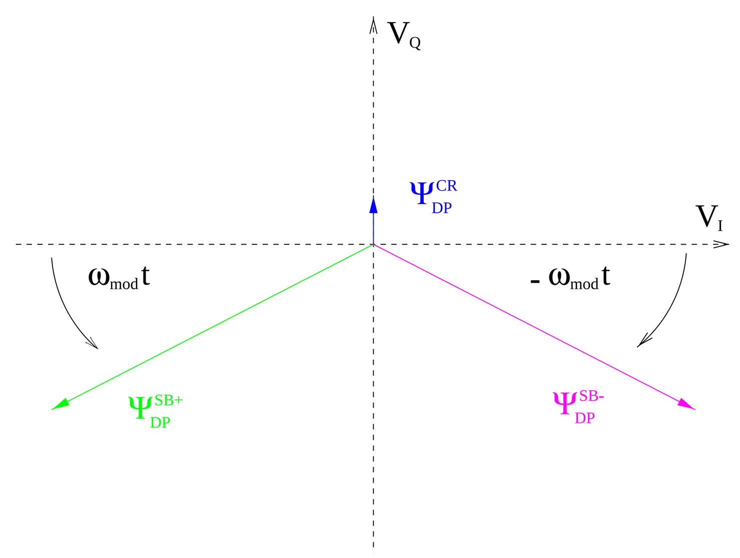
A reference frame for detecting the variation of the phase

$$\begin{aligned}\Psi &= \Psi_l e^{i\Gamma\cos\omega_{mod}t} \\ &\simeq J_0(\Gamma)\Psi_l + iJ_1(\Gamma)\Psi_l e^{i\omega_{mod}t} + iJ_1(\Gamma)\Psi_l e^{-i\omega_{mod}t} \\ &= \Psi_0^{CR} + \Psi_0^{SB+}e^{i\omega_{mod}t} + \Psi_0^{SB-}e^{-i\omega_{mod}t}\end{aligned}$$



$$\begin{aligned}P_{DP} = |\Psi_{DP}|^2 &= |\Psi_{DP}^{CR}|^2 + |\Psi_{DP}^{SB+}|^2 + |\Psi_{DP}^{SB-}|^2 + \\ &2\Re[(\Psi_{DP}^{CR}\Psi_{DP}^{SB-*} + \Psi_{DP}^{SB+}\Psi_{DP}^{CR*})\exp(i\omega_{mod}t)] \\ &+2\Re[\Psi_{DP}^{SB+}\Psi_{DP}^{SB-*}\exp(2i\omega_{mod}t)]\end{aligned}$$

For the "ideal" configuration

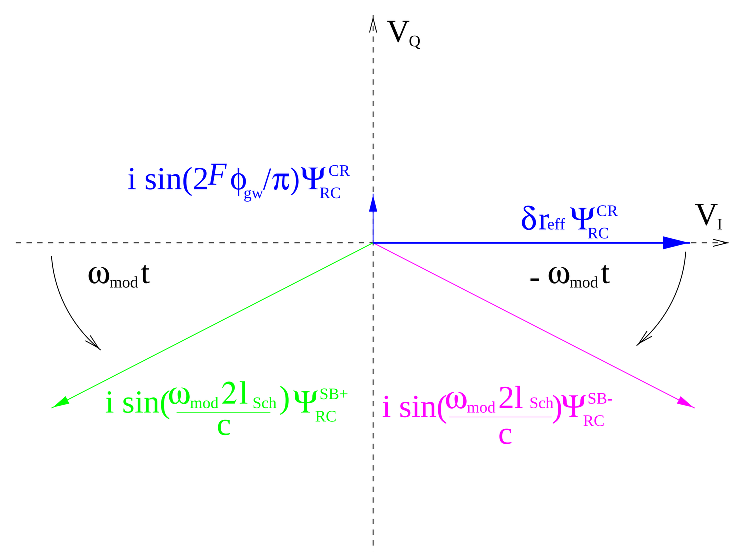


$$V_I = \int_0^T \frac{P_{DP}\cos\omega_{mod}t}{T}dt = \Re[\Psi_{DP}^{CR}\Psi_{DP}^{SB-*} + \Psi_{DP}^{SB+}\Psi_{DP}^{CR*}]$$

$$V_Q = \int_0^T \frac{P_{DP}\sin\omega_{mod}t}{T}dt = -\Im[\Psi_{DP}^{CR}\Psi_{DP}^{SB-*} + \Psi_{DP}^{SB+}\Psi_{DP}^{CR*}]$$

$$D.C. = \int_0^T \frac{P_{DP}}{T}dt = |\Psi_{DP}^{CR}|^2 + |\Psi_{DP}^{SB+}|^2 + |\Psi_{DP}^{SB-}|^2$$

In a "realistic" interferometer



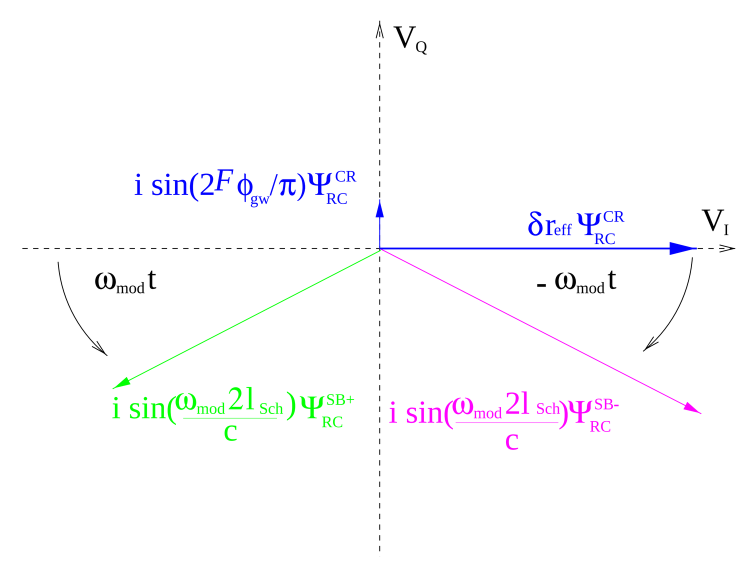
- Requirements for limiting any variation equivalent to a phase change due to some gravitational signal

  $\tilde{x}(10Hz) \leq 10^{-19}m/\sqrt{Hz}$ TMs displacement noise
- laser frequency and amplitude must be stabilized to reduce fluctuations

  relative intensity noise $\leq 10^{-9} - 10^{-8}/\sqrt{Hz}$

  laser frequency noise $\delta\nu_0 \leq 10^{-7}Hz/\sqrt{Hz}$

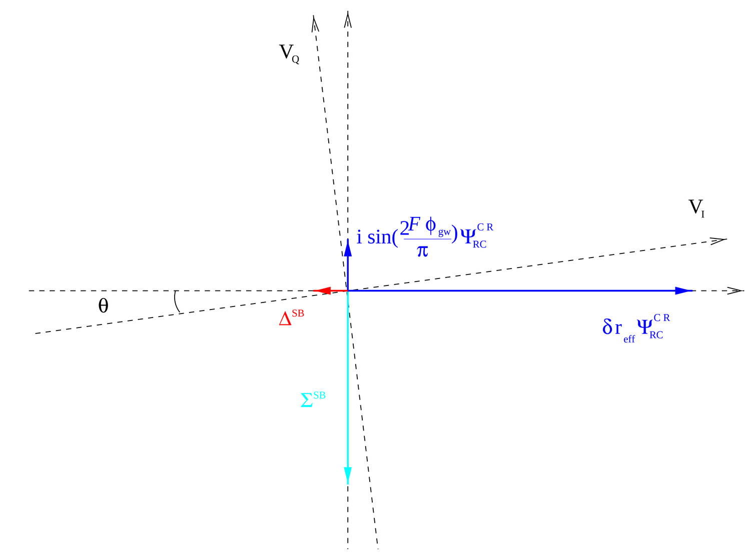
Any difference in amplitude between the two sidebands may couple with the carrier light that exits the dark port because $\delta r_{eff} \neq 0$.



If the contrast defect is not perfect, the amount of carrier light at the dark port may be quite large and even for a small sideband amplitude imbalance, the magnitude of the in-phase signal V_I can be comparable or larger than V_Q.

Even if the magnitude of the in-phase signal can become relevant and influence the gravitational wave channel, if they are completely separated and there is no mix at all between them, there is no impact on the information contained in V_Q. The angle θ is of the order of a few degrees and it is empirically tuned. In this case the gravitational wave channel would become

$$V_Q = \sin(\frac{2\mathcal{F}\phi_{gw}}{\pi})\Psi_{RC}^{CR}\Sigma^{SB}\cos\theta + \delta r_{eff}\Psi_{RC}^{CR}\Delta^{SB}\sin\theta$$



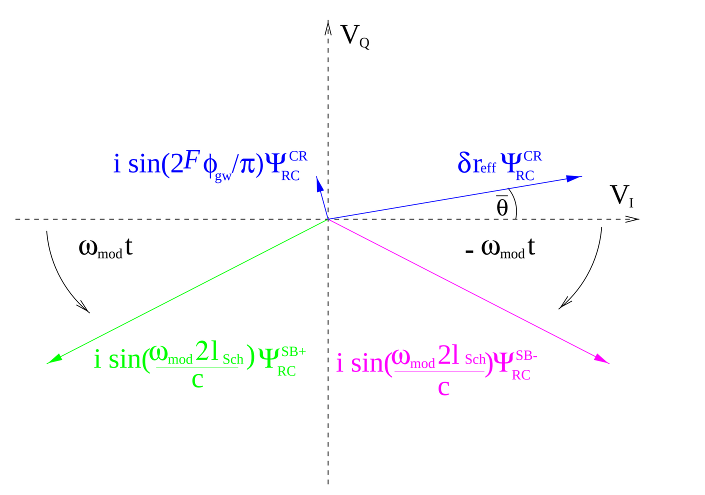
Mix problems due to a common phase shift affecting the sidebands

In an ideal interferometer the modal content of the carrier and the sidebands would be the same.

If it is not:

1. the sidebands accumulate a phase shift corresponding to the discrepancy between the Gouy phases
   $$\theta_G^{CR} \neq \theta_G^{SB\pm}$$
2. the amplitude of the sidebands is still the same only if all the cavity lengths are integer multiples of $\lambda_{mod}/4$. This macroscopic condition implies that the resonant curves of the sidebands are overlapped and the gain is the same for any microscopic tuning of the longitudinal degrees of freedom.

Contribution of the contrast defect to the gravitational signal



$$V_Q = \sin(\frac{2\mathcal{F}\phi_{gw}}{\pi})\Psi_{RC}^{CR}\Sigma^{SB}\cos\bar\theta + \delta r_{eff}\Psi_{RC}^{CR}\Sigma^{SB}\sin\bar\theta$$

$$\bar\theta \simeq \frac{\theta_G^{SB\pm} - \theta_G^{CR}}{1 - r_{RM}\cos(\frac{\omega_{mod}2l_{Sch}}{c})} \qquad (1)$$

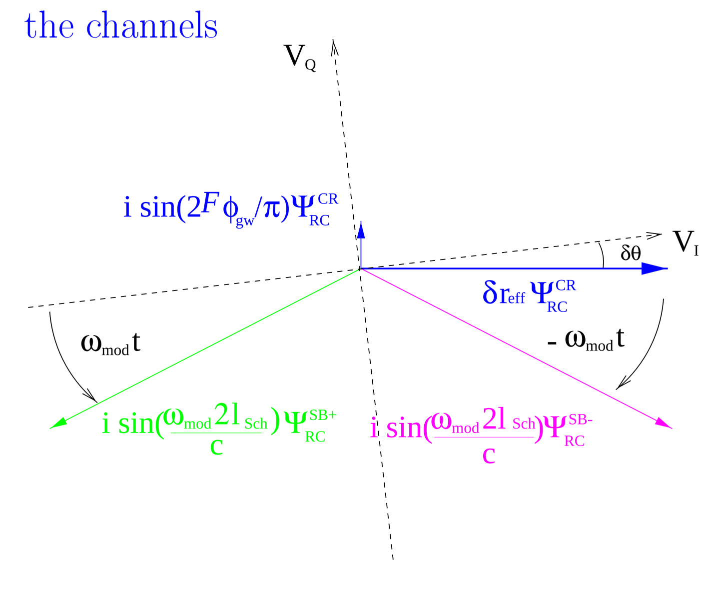
Mix problems due to the effective length of the recycling cavity

The ideal case would be maximum sideband power stored in the recycling cavity when the carrier resonates.

Some problems arise when:

1. the recycling cavity does not satisfy the macroscopic condition for its common length so that the sidebands accrue same and opposite phase shift when the carrier is on resonance. The sideband fields are complex conjugate and the gain is the same.
2. the sideband power is not the same in the two branches of the Michelson interferometer, that is the effective reflectivities from the two arms are not the same. Because of the Schnupp asymmetry the common length is a weighted sum of the two lengths

Partition of the gravitational wave signal between the channels

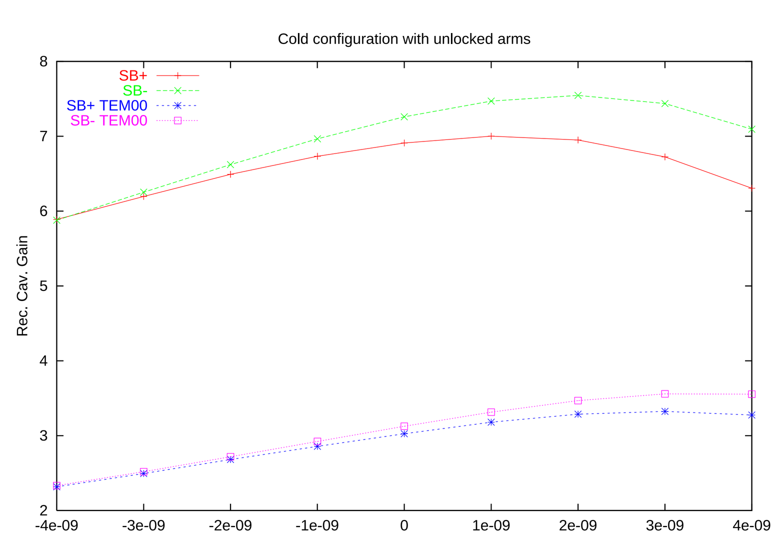


$$V_Q = \sin(\frac{2\mathcal{F}\phi_{gw}}{\pi})\Psi_{RC}^{CR}\Sigma^{SB}\cos\delta\theta$$

$$V_I = \sin(\frac{2\mathcal{F}\phi_{gw}}{\pi})\Psi_{RC}^{CR}\Sigma^{SB}\sin\delta\theta$$

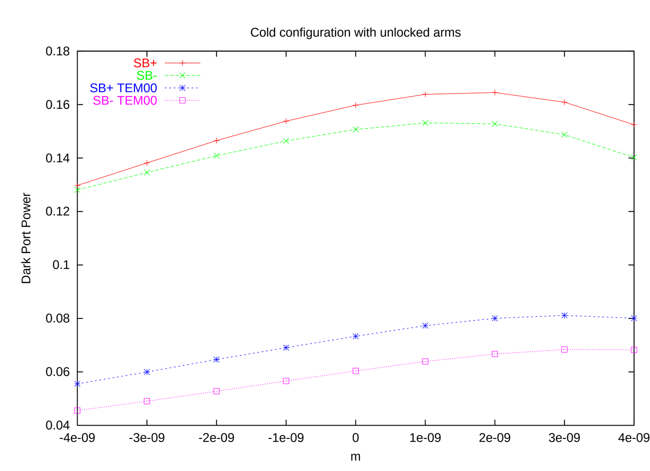
$$\delta\theta \simeq \frac{\frac{\omega_{mod}l_{\pm}}{c}}{1 - r_{RM}\cos(\frac{\omega_{mod}2l_{Sch}}{c})} \qquad (2)$$

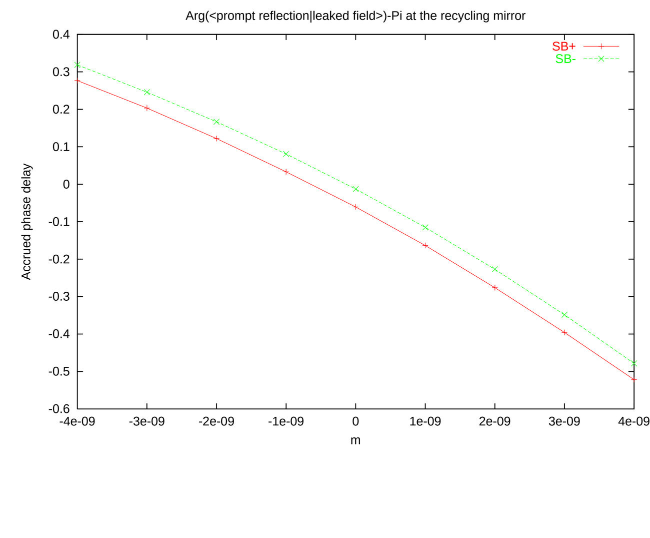
The resonant curves of the sidebands for one Ligo interferometer



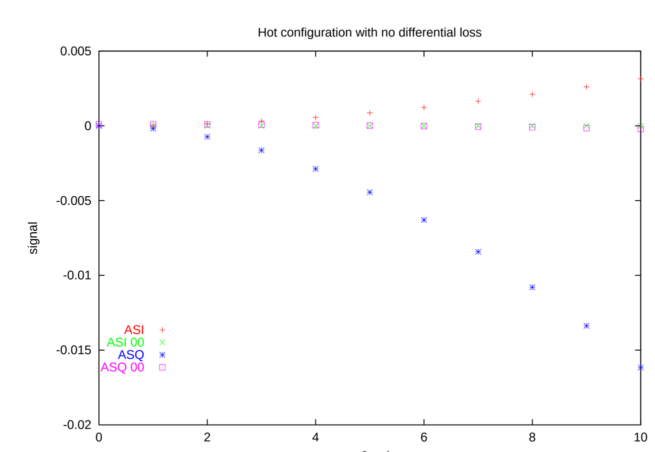
This graph shows a typical situation: the resonant curves of the sidebands are shifted and split because of (1) and (2) respectively. The plotted data correspond to the sideband power versus an offset introduced for exploring the resonant curves.

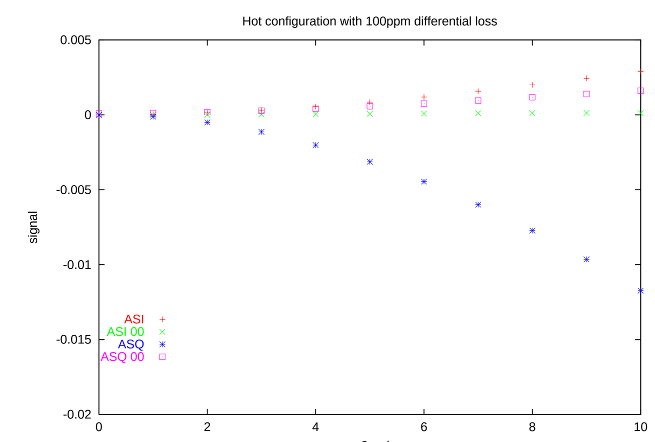
Power at the dark port and round-trip phase versus length tuning





If alignment is not perfect
the demodulated signals are affected





Conclusions and recommendations

- the sensitivity to differential length changes of the long arm cavities may be corrupted by the behaviour of the sidebands in real interferometers. In the ideal case the sideband gain would be maximum at the working point. If the interferometer is not perfectly balanced and matched, the gravitational signal V_Q is affected. The basic mechanism can be interpreted in terms of resonant curves: the carrier and sideband peaks occur for different longitudinal tuning of the recycling cavity.
- when higher order modes are involved, because of mirror imperfections or misalignment, the longitudinal degrees of freedom are adjusted to minimize the impact on the fundamental mode. As a consequence any change in V_I and V_Q due to transversal perturbations is mainly due to parasitic modes. In order to reduce those variations a mode-cleaner should be designed for the output port.