Acoustic properties of a hollow sphere for gravitational wave detection

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Abstract: We describe some experimental work on a small prototype of a hollow sphere, aiming at assessing the feasibility of such a resonator as a third generation g.w. resonant detector. We measured the resonant frequencies and quality factors of the spheroidal quadrupolar modes of a welded hollow sphere. The frequencies are found where expected, and the Q values were degraded by 20-60 % with respect to the bulk sphere.

1 Introduction

Resonant detectors of gravitational waves (g.w.) are reliable machines that have shown to operate for long uninterrupted periods of time. Cryogenic and ultracryogenic bars have indeed carried out long term observation with meaningful sensitivity. Spherical antennas [1] can represent the next generation of such detectors, as they can offer, with respect to a cylinder of the same linear dimension, larger mass (and therefore cross section) and omnidirectionality: the MINIGRAIN [2] group at Leiden University (NL) has been carrying out, over the last several years, most of the pioneering experimental work for the feasibility of a large scale spherical antenna. Indeed, an omnidirectional resonant detector can complement observations by large interferometers. Two choices are given, in this respect: explore higher frequencies than those where interferometers are most sensitive or insist on the same frequency band ($f \leq 1kHz$) to correlate interferometer information with data gathered on a different physical principle. The resonant frequency of the first spheroidal quadrupolar modes is roughly $V_s/2R$, where $V_s$ is the sound velocity of the material and R the sphere radius: for a 3m diameter sphere of Al5056 or CuAl this results in about 1 kHz. In some cases, one may want a lower frequency of sensitivity without dealing with an unpractically large resonator. A hollow sphere [3] can represent an interesting solution: the resonant frequency of its lowest quadrupolar modes can indeed be chosen with more flexibility with respect to the dimensions, simply by selecting appropriate values of internal and external diameter. Recent theoretical and numerical work [4] on the sensitivity of a hollow sphere confirms the interest in this kind of detector.
Further advantages of a hollow sphere are:

- A hollow sphere can be more easily fabricated in large dimensions, by welding together either thick, curved plates or superimposed rings of the chosen material: the fabrication is then reduced to a quasi-two-dimensional process. The loss in mass is marginal (≤ 25% ) as long as the thickness \( t \equiv R_{\text{ext}} - R_{\text{int}} \) exceeds one third of the outer radius.
- A hollow sphere can be cooled more easily: the shell thickness \( t < R_{\text{ext}} \) is cooled at the same time from the external and the internal surface: in this way we effectively reduce the thermal resistance and consequently the cooling time especially to ultralow temperatures.
- The second quadrupolar modes \{n=2, l=2, m= -2,...,2\} of a spherical resonator have a significant sensitivity to g.w. . In a hollow sphere their resonant frequency, and their cross section can be selected by proper choice on the ratio \( r \equiv R_{\text{int}}/R_{\text{ext}} \). It is then possible to position two different spectral windows of observation at frequencies of interest or choose an "aspect ratio" such to provide equal cross section at the two frequencies. In all cases this corresponds to an effective widening of the bandwidth of observation of a resonant detector.

Although most experimental aspects of a spherical detectors have been analyzed and solved in recent years [2], a hollow sphere proposes several new practical challenges and problems that need be studied and question that need be answered. In this work we have tried to address the following issues:

- A hollow sphere cannot be suspended by its center of mass: will a surface suspension affect the mechanical properties of the resonator ?
- The welded joints in a solid body affect its normal modes of vibration ? Does the welding discontinuity represent an obstacle for the elastic waves ? and if it does not:
- To what extent does the welded joint increase the mechanical losses for the vibrational modes of interest ? Indeed, it is well known that a high quality factor of the mode oscillations is required to achieve best sensitivity in ultracogenic detectors.

In order to find answers to these questions, we began experimenting with small metal spheres: after some preliminary tests with spheres of Aluminum alloy, \( R_{\text{ext}} = 25\, \text{cm} \), we focused on a sphere made of CuAl 6% , the same material of the Minigrail detector, with \( R_{\text{ext}} = 15\, \text{cm} \). Interesting enough, despite the different dimensions, the two samples have comparable mass (about 14 kg) and resonant frequency (about 13 kHz), due to the higher density of CuAl \( (\rho_{\text{CuAl}} = 8 \times 10^3 \, \text{kg/m}^3 \simeq 3\rho_{\text{Al}}) \).

2 The benchmark: measurement on a bulk sphere

A hollow sphere can be produced in at least two ways: by fusion or by welding parts together. Fusion with a melting core is probably the most straightforward method, as it produces a resonator without seams, and represents, for a small sample, no serious challenge. nevertheless we chose to investigate the welding of two half spheres for a twofold reason:

First, we are interested in exploring the feasibility of a large sphere, that would most probably be fabricated out of plates, as discussed above. Moreover, in this way we are able to compare the internal dissipation (measured by the Q of the oscillator) of a hollow sphere...
with that measured, before machining, on the bulk sample of the same material, size and instrumentation: in this way we are able to evaluate the Q reduction due to welding.

We started out with characterizing the bulk resonator from which we eventually derived the hollow sphere: it is a sphere in CuAl 6% (94 % Copper, 6% Aluminum), 15 cm in diameter, that was previously used and measured in Leiden University [5]. These measurements, as all those reported here, were made using two piezoelectric ceramics (PZT) glued on the sphere surface, at polar angles $\theta = \pi/4; \phi = \pm \pi/4$. Occasionally, we also used small accelerometers and impulsive excitation provided by a coil activated "hammer". As these auxiliary sensors and actuators cannot be used in cryogenic enviroment (due to excessive heat dissipation), the relevant data were in the end taken by exciting with one PZT and reading out with the other.

In order to suspend the sphere, we drilled a hole across its diameter, with two different sizes (see fig 1): 5 mm diameter from one pole to the sphere center, 6 mm diameter from the center to the other pole. The hole was tapped M7 at the wider end and near the center, making it possible to fasten a suspension cable both near the the sphere center of mass and on its surface. Eigenfrequency and Q measurements, carried out at room temperature in both configurations and shown in fig. 1, show no significant degradation due to surface suspension. These results are consistent with those obtained in ref. [5], down to ultracryogenic temperatures, but with a somewhat different suspension.

![Schematic of the sphere suspension](image)

**Figure 1:** Left: schematic of the sphere suspension: the hole is tapped both near the center and on the surface. These two different suspensions have the same influence on the quality factor of quadrupolar modes of a bulk sphere, as shown on the right.

This resonator has its first quadrupolar modes at $f_{\{n=1,l=2\}} = 13313Hz$. The Q values we measured, consistent with previous measurement done in Leiden, were spread around $10^5$ at room temperature and just below $10^6$ at liquid Helium temperature (4.2 K). Larger resonators, like Minigrail, have shown Q values up to ten times larger: these lower values can be due to several possible causes: insufficient isolation of the cryostat, larger surface to volume ratio, heavier influence of the suspension and transducers for a smaller oscillator, small metallurgical differences in the alloy or in the aging history of the sample. These
values represent the benchmark to which we must compare the results later obtained with
the hollow sphere.

3 Fabrication

Once the bulk sphere was properly characterized, we split it in two equal parts with wire
EDM machining, that removed the smallest amount of material (the cut was about 0.5 mm
wide), in order to minimize deviations from sphericity after reassembling the two halves.
Both hemispheres were then carved out and reduced to shells with 22 mm thickness, i.e.
r = 0.707.

We considered three methods of welding the two half spheres:

• Electron Beam Welding (EBW) Although high penetration welding is possible and common
on pure Copper, CuAl did not perform as well: a test weld on a small sample proved
to be brittle, uneven, full of cracks and with a penetration of few millimeters. Besides,
Italian welding companies would not provide joints thicker than a few cm: this technique
is not therefore exportable to a large size (R_{ext} > 1m,) sphere, because they can’t provide
penetration depth of the order of t ≥ 0.3m.

• Diffusion Welding it is a special welding technique, where no foreign material is interposed
between two mating surfaces. The native alloy, brought to a temperature close to the melting
point, diffuses across the boundary, hopefully recreating the same metallurgical bonds of the
bulk. This approach appears promising with respect to preserving the Q of the resonator,
as no discontinuity is met by the elastic wave traveling in the solid.

We carried out some tests on a hollow cylinder with the same radial thickness t = 22mm
we had designed for the hollow sphere: on this simpler geometry it is straightforward to
verify whether the bond between two shorter cylinders recreates one long elastic cylinder:
the first longitudinal mode is simply related to the length (f_1 = V_s/L) and can be identified
unambiguously. The two parts were kept for 1 hour under mechanical pressure at 1020 °C,
i.e. 50 degrees below the melting point of CuAl, and then allowed to cool slowly in vacuum.
Results at room temperature were encouraging for this test: both resonant frequency and
Q of the first longitudinal mode turned out to be virtually unaffected by the processes of
cutting and welding. However, when the sample was shock-cooled in liquid Nitrogen, its Q
was irreversibly degraded, most probably due to the insurgence of a crack in the weld.

• Silver brazing We therefore turned to the traditional technique of oven brazing with a thin
(~ 50μm) layer of silver based filler. The preliminary tests on the cylinder were passed with
full satisfaction, so we proceeded to brazing the two spherical shells.

4 Experimental Results

4.1 Resonant frequencies

The hollow sphere so produced was extensively tested both at room temperature and in
cryogenic conditions. Cryogenic tests were carried out in a cryogenic facility in the laboratory
of Tor Vergata University especially built, with due attention to isolation from external mechanical disturbances, in order to test mechanical resonators down to 4.2 K. We can summarize the experimental results as follows:

The quadrupolar modes of vibration of the sphere can be easily identified and are found clustered around the expected frequency. The resonant frequencies of the quintuplet at room temperature and at 4 K are shown in fig. (2). From these measurements we deduce three important points:

![Graph](image)

Figure 2: The eigenfrequencies of the first quadrupolar quintuplet \( f_{(1,2)} \) of the hollow sphere as measured at room temperature, at 77 K and at 4.2 K. On the right (300 K), the thicker (black) line represents the expected value, based on dimensions and material properties.

1. The elastic waves propagate across the bonding discontinuity of the resonator without measurable effect on the resonances; although we had no provision for visualizing the mode shapes, we can infer that the quadrupolar modes were not affected by the presence of a layer of bonding material: in other words, welding reconstructs an elastic sphere.

2. The resonant frequency of a hollow sphere quadrupolar modes is predicted [3] to be a monotonically decreasing function of the radii ratio \( r \): we have found \( f_{(1,2)} = 7537Hz \) i.e. exactly where the theory predicts it for \( r = 0.7 \), therefore validating the calculation of Lobo et al.

3. By carving a large internal cavity we removed less than 35% of the mass (from 14.4 to 9.4 kg). This would bear little consequence on the sensitivity of a real detector. However the resonant frequency of operation is lowered to about half (57% in our case) of the initial value, showing that this is an effective way to tune the resonator to a given frequency.
We also performed a detailed finite element analysis of both the bulk and the hollow sphere, where the suspension was modeled and taken into account. The resulting computed eigenfrequencies agree very well (see fig.3) with the measured values, and give a satisfactory quantitative account of the splitting of four modes out of the five considered. The splitting of the quadrupolar quintuplet is mostly due to the suspension hole that crosses the sphere on a vertical diameter and to the suspension itself.

![Graph showing resonant frequencies](image)

Figure 3: The resonant frequencies of the five lowest quadrupolar modes: the black line represents the value predicted by the theory of ref. [3], where the degeneracy is not removed and no splitting is predicted. The green triangles show the prediction of the Ansys finite element calculation and the red circles are the experimental values.

Also the second quadrupolar modes where found at the expected frequencies. Unfortunately, their resonant frequency, \( f_{(2,2)} = 28kHZ \) was too high to couple well with our measuring apparatus and so detailed information could not be obtained.

### 4.2 Internal friction and quality factors

Q measurements were carried out on the quadrupolar modes at temperatures between 300K and 4.2K. The results, showing the typical increase in Q values with decreasing temperature, are shown in fig.(4).

The measured Qs are consistently lower than those previously measured on the bulk sphere: in fig.(5) we show a comparison of the values measured at room temperature, 77 K and at 4.2 K.

We observe that at room temperature, as well as at 77 K, the Q loss is within a factor 2, with a typical reduction, averaged among the 5 modes, to about 65 % of the values of the bulk sphere.

At liquid Helium temperature the effect is more relevant: the measured values are between 0.8 – 3.5 \( \cdot 10^5 \), with an average reduction to 35 % of the bulk values. As the quality factors increase only marginally from 77 to 4 K, we can deduce that some dissipation mechanism limits them at the \( 10^5 \) level. This loss mechanism could be either due to losses in the welded joint or to a heavier influence of the suspension on this geometry. We cannot rule out, however, that this level of losses be intrinsic, i.e. due to the geometry of the hollow sphere.
and, for example, to its higher surface to volume ratio or to its lower resonant frequency. Our tests did not provide sufficient information to discriminate between these, or other hypotheses.

Figure 4: Quality factors of the quadrupolar modes vs. temperature.

Figure 5: Comparison, at 300 K, 77 K and at 4.2 K, between the measured Q values of the quadrupolar modes of a bulk and hollow sphere. The larger dissipation in the hollow sample becomes more evident as the Q grows higher at low temperature.
5 Conclusions and perspectives

We have investigated some experimental tests regarding construction and operation of a hollow sphere as a third generation detector. We have verified that, in a bulk sphere, surface suspension does not degrade the quality factor of the first quadrupolar modes. We tested various methods of bonding two carved half spherical shells to create the hollow resonator. In our tests the traditional silver brazing gave the best result, and we used it to fabricate our resonator. The resonant frequency of the first quadrupolar modes of oscillation was found exactly where the theory (that had never been verified before) predicts it, thus showing that a welded bond does not interfere with the propagation of elastic waves. We have measured the quality factors of the quadrupolar modes from room temperature down to 4.2 K and found them to be limited to about $10^3$ and consistently lower than those of the bulk sphere. The cause of this extra source of dissipation is to be identified, although the welded joint is the natural suspect.

Future work will include a systematic search of the source of extra dissipation. We also plan to further investigate the technique of diffusion welding, that appears in principle the most promising: the problems encountered in the first tests could be overcome by modifying the procedure, i.e. by heating the sample even closer to the melting point, in order to improve the bonding.

The results of our tests show that a hollow sphere, proposed years ago as a versatile resonant g.w. detector, is a viable solution.

References